Rocket Performance Prediction Technique

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A simple, semi-empirical performance correlation/prediction technique applicable to gaseous and liquid propellant rocket engines is presented. Correlations were attained by "adjusting" the computation of the gaseous mixing of an unreactive, coaxial jet using a correlation factor, F, which resulted in prediction of the experimental combustion efficiency for each firing. The technique was successfully applied to Rocketdyne, Aerojet, TRW, and Bell Aerospace gaseous $H_2/0_2$ rocket engines utilizing coaxial, triplet, trislot, premix, and reverse flow injector elements, and to Bell's 6000 lb-thrust orbital maneuvering and 600 lb-thrust reaction control engines, which utilize triplet and unlike doublet injector elements, respectively, and liquid monomethylhydrazine and nitric oxide propellants. Test data from over 100 firings, representing a wide range of engine sizes and flow conditions, were compressed when the correlation factor, F, times the chamber length, L, divided by the "effective" radius, R_I, was plotted vs the ratio of the injection velocity of the fuel divided by the injection velocity of oxidizer. The resulting correlations are useful for comparing the effectiveness of different injectors at the same velocity ratios (which are proportional to the mass flux ratios), for predicting optimum operating conditions for a given injector geometry, and for assessing the consistency of test data. Before liquid rocket performance predictions can be made with confidence, additional liquid rocket data, covering a wider range of conditions, must be correlated.

Nomenclature

ΣA = total injection area for a particular propellant, ft^2
D_ρ = empirical density parameter ($\equiv \rho_u/\rho_{min}$)$^{1.5}$
EL = number of injection elements
F = correlation factor ($\equiv \epsilon_\eta/\epsilon_{model}$)
f = empirical length parameter ($\equiv 0.5+0.005\,\bar{z}; \bar{z}<100$, and $1.0; \bar{z}\geq 100$)
G = total mass flow rate of gaseous propellant measured at injector, lbm/sec
L = length of combustion chamber (injector to nozzle throat), ft
MW = molecular weight, lbm/lb-mole
$\dot{m}$ = computed mass flow rate, lbm/sec
P_c = chamber pressure, lbf/ft^2
Pr_T = turbulent Prandtl number
R = universal gas constant, 1545 ft-lbf/lb-mole-°R
R_I = "effective" radius of central oxidizer injector, defined in Eq. (9), ft
r = radial coordinate, ft
Sc_T = turbulent Schmidt number
T = temperature, °R
U = axial velocity, fps
V_R = velocity ratio ($\equiv U_{H_2}/U_{0_2}$)
V'_R = pseudo velocity ratio defined for liquid propellants assuming they are completely vaporized prior to injection
Y = mass fraction
z = axial coordinate, ft
$\bar{z}$ = normalized axial coordinate ($\equiv z/2R_I$)
ϵ_{model} = turbulent (eddy) viscosity predicted by Eq. (3), lbm/sec-ft
ϵ_η = turbulent (eddy) viscosity used in computation of η_{pred} ($\equiv F\epsilon_{model}$), lbm/sec-ft
η_{exp} = experimentally determined combustion efficiency
η_{pred} = combustion efficiency predicted using computer program
ρ = density, lbm/ft^3

Subscripts

e = external or freestream conditions
u = condition at velocity half radius where $\bar{U}\equiv 0.5\,(U_{max}+U_{min})$

Received March 27, 1972; subsequently presented as AIAA Paper 72-1074 at the AIAA/SAE 8th Joint Propulsion Specialist Conference, New Orleans, La., November 29-December 1, 1972; also presented at the 11th JANNAF Combustion Meeting, Pasadena, Calif., September 1974; revision received September 19, 1974. This work was supported by NASA Lewis Research Center, under Contract NAS3-16798, R. J. Priem, Project Manager. The authors are indebted to R. J. Priem, NASA-Lewis, and J. G. Skifstad, Purdue University, for many helpful discussions and suggestions concerning this Rocket Engine Performance/Prediction Technique and its presentation. They also are indebted to O. Fortune, Consultant, who programed and debugged the computer program used for the correlation, and to S. Feyl, Bell Electronic Data Processing, who made the program operational on the IBM 7090. They are especially indebted to L. Montanino, Bell Combustion Devices/Propulsion Systems and Components, who selected and assisted in reducing the performance data correlated herein.

Index categories: Combustion in Gases; Computer Technology and Computer Simulation Techniques: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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I. Introduction

As combustors become more complex and mission considerations require maximum delivered performance, the need for quantitative prediction of performance of advanced propulsion systems becomes increasingly important. In the past, the design and optimization of rocket and ramjet combustors has been accomplished primarily using trial-and-error procedures. Although this approach can be very costly and time consuming, the extremely complex processes—for example, injection, mixing, and combustion—that occur in a practical combustor, and their interactions, have defied detailed analysis leaving little alternative.[1] Recent success in the development of a simple empirical eddy viscosity model[2-4] suggested that it might be used in a semi-empirical correlation technique based on the rational assumption that the turbulent mixing of reactants is the rate controlling process in combustors employing diffusion flames. Reference 5 is an expanded version of this paper.

II. Analysis

Basis

The correlation technique was based on the assumption that turbulent mixing alone is the rate-controlling process; therefore, the rate of chemical reactions that occur in combustors was assumed to be very fast compared to the mixing rate. Since almost all practical combustors are fueled with extremely reactive propellants, this assumption appears plausable; making it permits engine performance, i.e., combustion efficiency, to be computed without considering the finite-rate chemical kinetics, and resulted in an enormous savings in computer time. A final simplification, which also appeared justified since only an empirical correlation was being sought, was that the stagnation temperature remains constant and equal to the initial (unreacted) propellant temperature before combustion, rather than using a computed temperature based on the extent of the mixing and reaction.

Of course, performance computed using a mixing model applicable to nonreactive coflowing free jets, could not be expected to predict actual rocket engine performance without modification. The approach selected herein was to empirically adjust the values of the eddy viscosity predicted using the mixing model of Ref. 2 (to be discussed) by multiplying it by a correlation factor, F, which remained constant in each performance calcualtion, that is, the value of the turbulent mixing coefficient (eddy viscosity) used for the performance calculation became

$$\epsilon_\eta = F\epsilon_{model} \tag{1}$$

The value of F was obtained by iteration for a particular set of performance data until

$$\eta_{pred} \approx \eta_{exp} \tag{2}$$

The value of η_{pred} was within 0.1% of the value of η_{exp} for all correlations reported herein.

Mixing Model

Details of the development of a simple empirical, mass defect, eddy-viscosity-type, turbulent mixing model was reported in Ref. 2. This model is

$$\epsilon_{model} = 0.018D_p(1+2f)\frac{\int_o^\infty |\rho U - \rho_e U_e| rdr}{r_u + (2R_1 - r_u)exp(-0.115\bar{z})} \tag{3}$$

Performance Computation‡

In the correlation procedure, the combustion efficiency was computed by considering only the nonreactive mixing of the H_2 and 0_2 propellants; no chemical reaction nor change in stagnation temperature was considered. In order to compute the combustion efficiency, η_{pred}, the assumption was made that whenever H_2 and 0_2 were mixed *locally*, the H_20 formed at that point could be computed directly from the limiting constituent. The total mass flow rate of H_20 that resulted from this assumption was determined by numerical integration at each axial station; its division by the maximum mass flow rate of water that could be formed when all the 0_2 was reacted (since H_2 always was in excess) yielded the predicted combustion efficiency, η_{pred}. As will be demonstrated, these simplifying assumptions, while precluding computation of detailed flow conditions such as local concentration or velocity, do *not* adversely affect the prediction of combustion efficiency.

The local mass fraction of H_20, Y_{H_20}, that would have occurred as a result of mixing was computed as

$$Y_{H_20} = 9/8\,Y_{0_2}, \text{ when } Y_{0_2}/Y_{H_2} \leq 8 \tag{4a}$$

‡A typical case required approximately 90 sec of IBM 360-65 computer time.

$$Y_{H_20} = 9Y_{H_2}, \text{when } Y_{0_2}/Y_{H_2} > 8 \tag{4b}$$

For the cases considered, in which H_2 was always in excess, the maximum quantity of water formed was proportional to the initial flow rate of 0_2; that is, all the 0_2 would ultimately react to form H_20, or

$$(\dot{m}_{H_20})_{max} = 9/8(G_{0_2}) \tag{5}$$

Therefore, the overall combustion efficiency at any axial station was computed as

$$\eta_{pred} = \frac{2\pi}{(\dot{m}_{H_20})_{max}} \int_0^{r_\infty} \rho U Y_{H_20} rdr \tag{6}$$

where r_∞ is the freestream boundary which defines the extent of the shear layer, i.e., mixing region.

Flow Conditions

The injection velocities for the H_2 and 0_2 were computed using the continuity equation and the perfect gas law, e.g., U_{0_2}, the injection velocity of the gaseous 0_2 propellant was computed from the relation

$$U_{0_2} = \frac{(G_{0_2})(R)(T_{0_2})}{(P_c)(MW_{0_2})(\Sigma A_{0_2})} \tag{7}$$

A similar computation was made to calculate the injection velocity of the gaseous H_2, U_{H_2}. The velocity ratio, V_R, was simply computed as

$$V_R = U_{H_2}/U_{0_2} \tag{8}$$

The "effective" radius R_1 was computed from the relation

$$R_1 = \left[\frac{(\Sigma A_{0_2})}{\pi(EL)}\right]^{1/2} \tag{9}$$

III. Results and Discussion

Computations

The predictions of combustion efficiency were made using a standard finite difference computational technique, similar to that described in Ref. 6, in which the governing shear layer equations (continuity, species diffusion, momentum, and energy equations) were solved in the von Mises coordinate system. The calculations were always begun assuming slug (step) profiles and $Sc_T = 0.7$, $Pr_T = 1.0$. The initial Y_{0_2} profile used to start the computation ($z=0$) for Rocketdyne Case 12H,[7] is plotted in Fig. 1 along with the Y_{0_2} profiles computed at intermediate and final stations of 0.5, 2.5, and 5 in., which were computed using the unreactive mixing calculation discussed in Sec. II. Note, that for the binary $H_2/0_2$ system considered, $Y_{H_2} \equiv 1\text{-}Y_{0_2}$, so that the corresponding Y_{H_2} values can be obtained from the plots as well. The velocity profiles at the same stations are plotted in Fig. 2. The initial step velocity profiles at $z=0$ are simply the bulk mean velocities computed using Eq. (7). The Y_{H_20} profiles computed using either Eqs. (4a) or (4b) are plotted in Fig. 3 at the same axial stations. As explained in Sec. II, the water was merely computed from either H_2 or 0_2 concentrations (whichever was the limiting reagent), and played *no* role in the mixing calculation; of course, the Y_{H_20} profiles were required for the computation of the combustion efficiency, η_{pred}, using Eq. (6). Although more sophisticated computations, e.g., including the finite rate chemistry, are within the capability of the computer program, the success achieved with the very simple approach suggested that expenditure of additional computer time was unwarranted—as long as only the combustion efficiency was to be predicted rather than details of the flow.

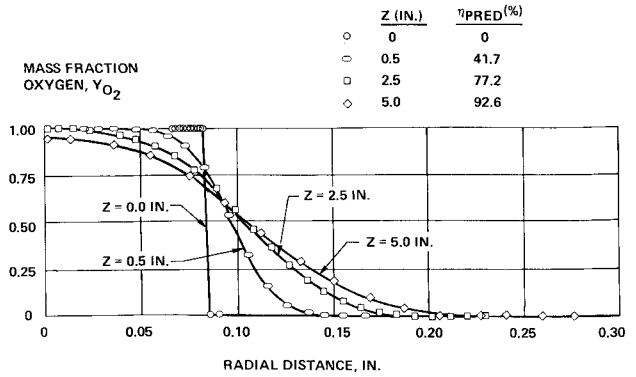

Fig. 1 Mass fraction oxygen at various axial stations computed using ϵ_η of Fig. 4; Rocketdyne coaxial injector test 12H.[7]

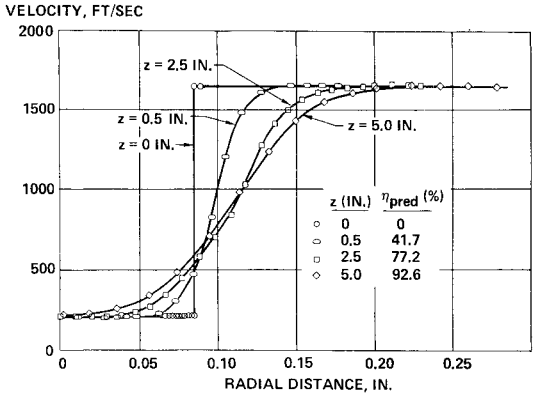

Fig. 2 Velocity profiles at various axial stations computed using ϵ_η of Fig. 4; Rocketdyne coaxial injector test 12H.[7]

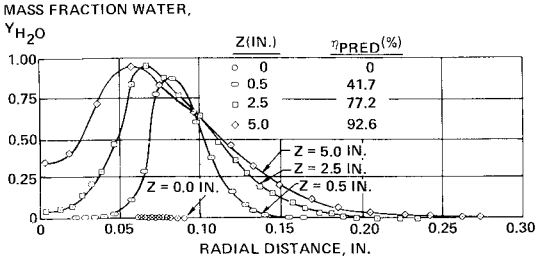

Fig. 3 Water profiles at various axial stations computed using Eq. (4); Rocketdyne coaxial injector test 12H.[7]

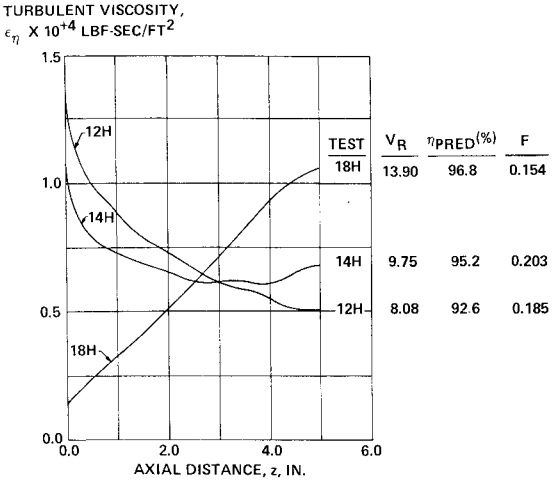

Fig. 4 Value of turbulent (eddy) viscosity used in η computation for Rocketdyne coaxial injector test 12H, 14H, and 18H.[7]

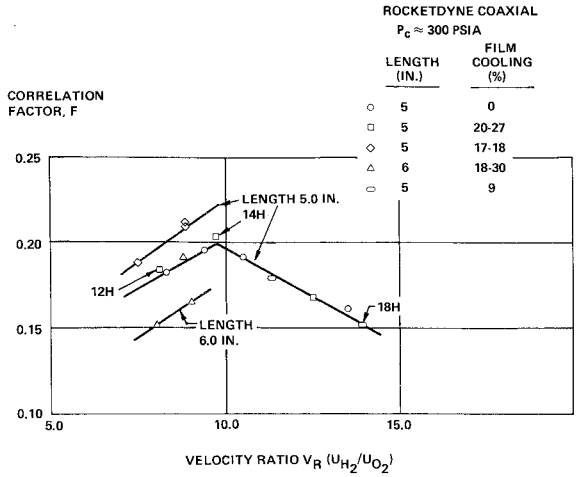

Fig. 5 Correlation of Rocketdyne coaxial injector performance data.[7]

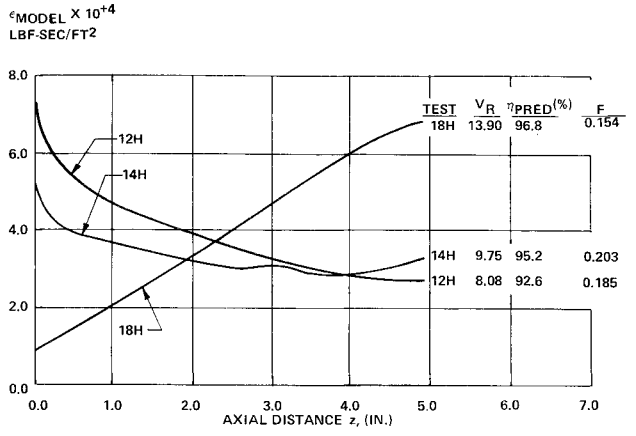

Fig. 6 Comparison of predictions of mixing model, [Eq. (3)] for Rocketdyne coaxial injector tests 12H, 14H, and 18H.[7]

The $H_2/0_2$ mixing computations are well illustrated in Figs. 1-3. There is gradual mixing of both mass and mom n-tum (stagnation temperature remains constant). As mass mixing proceeds, the computed water level grows, and hence, η_{pred} continually increases [Eq. (6)]. Starting with a given set of initial conditions, there is a one-to-one correspondence between the Y_{H_20} profiles and η_{pred}. In the iteration to determine the appropriate value of F, the profiles at the final axial station, $z=L$, are "adjusted" until η_{pred} computed from Eq. (6) yields η_{exp} to within the desired tolerance, e.g., 0.1%.

Values of the turbulent viscosity, i.e., eddy viscosity, ϵ_η, used throughout the mixing computation for Rocketdyne Tests (12H, 14H, and 18H,[7]), are plotted in Fig. 4. Note that by assumption ϵ_η is *not* a function of radial position, and so is constant at each axial position. These particular tests cases were selected because they covered the entire V_R range of the Rocketdyne Coaxial Injector test data; test 14H is at the maximum of the curve of F vs V_R in Fig. 5. Results in Fig. 5 are typical of the correlations of all the data presented in Ref. 5. For comparison, values of ϵ_{model} computed directly from Eq. (3) are presented in Fig. 6. Of course, these values are directly proportional to the ϵ_η's plotted in Fig. 4 [Eq. (1)]; however, their relative magnitudes are shifted because the F's vary (as do the η's). In view of the erratic behavior of these mixing coefficients, it is very surprising that useful correlations, such as those presented in Fig. 5 were obtained; however, useful correlations have been obtained for every case to which the technique has been applied.

Correlation of Data

Application of the correlation/prediction technique to 100 gaseous $H_2/0_2$ rocket engine firings of Refs. 7 to 11, yielded smooth correlation curves when the correlation factor, F, [Eq. (1)] was plotted vs the velocity ratio, V_R, [Eq. (8)], similar to that presented in Fig. 5 for Rocketdyne Coaxial Injector.[5] Successful correlations were attained for five different types of injectors: Coaxial, Trislot, Triplet, Premix, and Reverse Flow. The fact that smooth continuous curves were obtained for all these data suggested that the technique would be useful for prediction as well as correlation. Further analysis (discussed in detail in Ref. 5) demonstrated that the correlation function, $F \cdot L/R_1$, (where F is the correlation factor, L is the chamber length from injector to throat, and R_1 is the "effective" radius of the central 0_2 injector element) when plotted vs V_R, was able to compress the data for all types of injectors. Figure 7 is the composite plot of the results for all the gaseous $H_2/0_2$ engines analyzed to date.

The role of the correlation function appears to be similar to that of the drag coefficient for flow around bodies of various shapes, which predicts the shapes that will exhibit the highest drag at a particular Reynolds number, i.e., set of flow conditions. The correlation function, $F \cdot L/R_1$, permits similar comparison of the relative effectiveness of various injector types at a particular V_R, because it is independent of injector geometry. The reverse flow correlations were surprisingly consistent with the others in Fig. 7, despite the fact that a single oxygen injector was used with R_1 28 times as large as in the TRW Triplet.

Discussion of Correlation

One unexpected result is the sharp break in the correlation function that occurs at V_R between 8 and 10, which very interestingly occurs near the overall stoichiometric ratio of 8 at which the $H_2/0_2$ completely reacts to form water. The most likely reasons for this behavior are:

1) Failure of the mass-defect eddy viscosity mixing model to adequately predict details of the turbulent mixing reacting flow. The model predicts mixing to be proportional to the mass defect integral, Eq. (3). It is unlikely that a complex phenomena such as turbulent mixing will increase linearly with mass deflect over the entire range of flow conditions from $V_R = 1$ to 25. For example, the density of the propellants at the injection station are proportional to their molecular weights (the molecular weight ratio is 16), and the velocity of the hydrogen is always greater than that of the oxygen ($V_R > 1$), so that a range of velocity ratios exists (near 16) for which very small mass deflect integrals will be computed over a considerable length of the chamber. The effect is demonstrated in Fig. 6 by the very low initial value of ϵ_{model} for test case 18H in which $V_R = 13.9$. Of course, the extent of the mixing that has occurred at a particular axial station influences both the local mean molecular weight (and hence the density) as well as the local mean axial velocity; therefore as shown in Fig. 6, ϵ_{model} varies with axial position sometimes increasing and sometimes decreasing depending on the initial conditions.

2) The hydrogen stream was assumed to be infinite in extent in the correlation calculation, which means that the velocity of the hydrogen stream at its outer edge (freestream condition) remains *constant* throughout the calcualtions. Clearly, such a condition does not exist in a thin coannular jet typical of a coaxial injector in which the velocity decays rapidly (and in which transverse pressure gradients exist), as demonstrated experimentally in Ref. 12. The persistence of this high-velocity outer stream, and hence Reynolds shear stresses (ϵ_{model}), in the computation causes the mixing to be over predicted; however, the factor F, compensates for this effect, since the performance of *actual* combustors (e.g., with thin annular hydrogen jets) is being computed.

Although the relative importance of these various effects is uncertain, it is not necessary to understand them in detail, since accurate predictions of the engine performance (combustion efficiency) is obtained with the existing correlation function. The fact that practical results can be attained without detailed understanding of all of the extremely complex physical processes involved is one of the major advantages of a practical semiempirical approach such as the one presented herein.

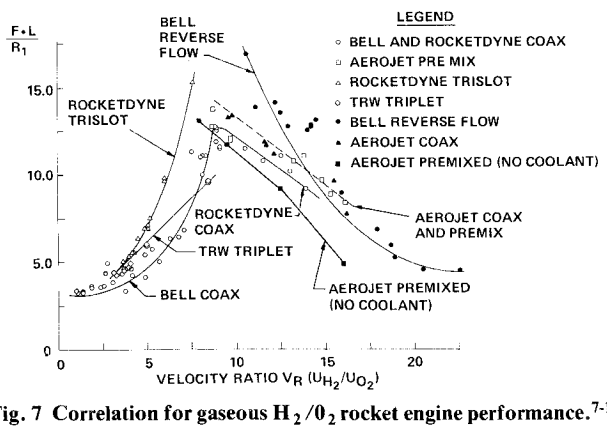

Fig. 7 Correlation for gaseous $H_2/0_2$ rocket engine performance.[7-11]

Evaluation of Design Parameters

The success attained with the correlation function when plotted vs the velocity ratio (Fig. 7), suggests that only those quantities explicitly included in these parameters are of major importance, and that all others are of only secondary importance in the prediction of gaseous rocket engine performance. Two such secondary effects are the chamber pressure and the film coolant level. Note, variables like the injector element spacing, chamber diameter, nozzle configuration, heat losses, etc., also do not appear explicitly in the correlations; however, their significance cannot be assessed directly because they are constant for each particular engine, i.e., there is no simple way of evaluating them. They undoubtedly influence the value of F. A test program designed specifically to investigate the secondary effects would be very beneficial. Because of the demonstrated insensitivity of the correlation technique to size, such a program could be conducted for the most part with small-scale engines. Those design parameters for which definite conclusions appear justified are summarized as follows.

1)Velocity ratio, V_R, has been demonstrated to be the most important parameter in the correlation (e.g., Fig. 7). Note that the mass flux ratio, $(\rho U)_{H_2}$and $(\rho U)_{0_2}$, is directly proportional to V_R and that the constant of proportionality is $(MW_{H_2})/(MW_{0_2})$ when the static pressure and temperature of each propellant is the same as the injection station. The fact that these ratios are important rather than the magnitude of the actual velocities is initially surprising. Examination of the mass-defect mixing model used in the correlation [Eq. (3)], and consideration of the species diffusion, axial momentum, and continuity equations yields the explanation. If the initial values of both U_{0_2} and U_{H_2} are increased (or decreased) by the same factor, V_R, will remain *constant*; however, the mass defect and hence ϵ_{model}, also will be increased (or decreased) by this identical factor (as long as the static temperature does not vary significantly). The continuity equation requires that the transverse derivative of the radial velocity, and hence the radial velocity itself, increases (or decreases) by this same factor, because the axial and transverse derivatives of the axial velocity, U, both increase (or decrease) by the factor. Therefore, when U_{0_2} and U_{H_2} are varied simultaneously holding V_R constant, every term in the species diffusion equation is multiplied by the factor (the turbulent mass mixing coefficient is equal to $\epsilon_\eta/0.7$), and identical concentration profiles, and combustion efficiencies, are

obtained for all species in each case. However, every term in the shear layer momentum equation (when $\partial P/\partial z = 0$) is multiplied by the square of the factor, which results in the velocity throughout the flowfield being increased (or decreased) by the factor. Computations demonstrated this explanation to be valid. Therefore, the velocity ratio, V_R, rather than the velocity of the individual streams is the critical parameter influencing the computation of η_{pred}. The fact that reasonable correlation of actual rocket test data was achieved with these computations, suggests that they adequately approximate the actual complex processes that occur in combustors employing gaseous fuels when used in conjunction with the empirical correlation factors, F.

2) The correlation function, $F \cdot L/R_1$, suggests that the ratio of the chamber length, L, to the "effective" central jet radius, R_1, is the critical geometric parameter in the design of gaseous fueled rocket injectors. This ratio implies that similar performance can be expected from "scaled" combustors of various sizes as long as this ratio remains constant. Of course, such scaling can be expected to apply only over reasonable ranges of combustor dimensions, i.e., those within the range of the test data correlated herein.

3) Chamber pressure, P_c, was shown to have little influence on the correlation factor, F.[5] The overall success of the correlation function, $F \cdot L/R_1$, (Fig. 7) implies there is no P_c effect, at least over the range of P_c's from 15 to above 450 psia. The H_2/O_2 kinetics are sufficiently fast so that such a pressure variation is not significant. The computations are independent of pressure because when pressure gradients are neglected within the combustion chamber, as they are in the correlation technique, the shear layer momentum equation and continuity equations, are both independent of pressure, when a mass-deflect mixing model and the perfect gas law are used to compute ϵ_{model}. Note, if the flow is assumed to be isoenergetic, the momentum equation is not completely independent of the static temperature (as it is of the pressure), since static temperature depends on the local velocity. However, for subsonic injection velocities, static temperature variation is small and density variations are caused primarily by the molecular weight variation.

4) The film coolant was shown to have little effect on the correlation.[5] In fact, only for the Aerojet premixed (no coolant) data in Fig. 7 does the Film Coolant Level appear to influence the value of F. As explained in Sec. II, all correlations were made neglecting the hydrogen fuel injected in the film barrier, focusing on only a "typical" injector element. Of course, if the combustor design were such that a significant portion of the film coolant were ingested into the combustion zone and reacted, this assumption would no longer be valid. Apparently, such an effect occurred in the Aerojet premixed injector.

In Fig. 7 the solid squares designated Aerojet premixed (no coolant) correlate more poorly than any of the other points. Interestingly, the Aerojet coaxial, and the Aerojet premix cases in which 20%-30% of the total hydrogen was injected as film coolant, all correlate reasonably well with a single straight line. Assuming no errors in the reported data, the reason for these film coolant effects appears to be that high levels of coolant increase mixing (yielding higher values of $F \cdot L/R_1$ at a given V_R) in the premixed injector geometry, by causing some of the coolant to react. In the premixed injector, H_2/O_2 is presumably mixed *prior* to injection and the likelihood of the hydrogen coolant reacting with the oxygen present in its immediate vicinity (near the wall) is obviously far greater than when each oxygen injector element is completely surrounded (and hence "protected") by an excess of hydrogen, as is the case for most of the other types of injectors. Based on the limited data correlated, the premix injector configuration does not appear to be as effective as more conventional configurations. This unexpected result must be substantiated before definite conclusions can be drawn.

Application to Liquid Rockets

The successful application of the semi-empirical correlation/prediction technique to gaseous H_2/O_2 rockets suggested that it might be applied as well to liquid rockets. Of course, in liquid/liquid rocket engines, droplet formation, injection, penetration, spreading, mixing, burning, and vaporization occur as well as gaseous mixing. Therefore, the magnitude of the "*equivalent mixing*" of a cold-flow, coaxial gas/gas injector element would be considerably smaller when it was used to model liquid/liquid injectors than when it was used to model gas/gas injectors for the same value of V_R, i.e., the correlation factor, F, will be smaller for liquid propellants than for gaseous propellants; however, the correlation/prediction technique still can be applied.

To demonstrate this capability, Bell Aerospace Reaction Control (RCE) and Orbital Maneuvering Engine (OME) test data were correlated; in each case, the fuel was monomethyl hydrazine (MMH) and the oxidizer was nitric oxide (N_2O_4). However, two very different types of injector configurations were used in these engines; the RCE elements were unlike doublets, while the OME elements were triplets [two MMH jets impinging (at 32°) on a central N_2O_4 jet].[13,14]

In order to demonstrate the applicability of the correlation/prediction technique to liquid propellants as sim-

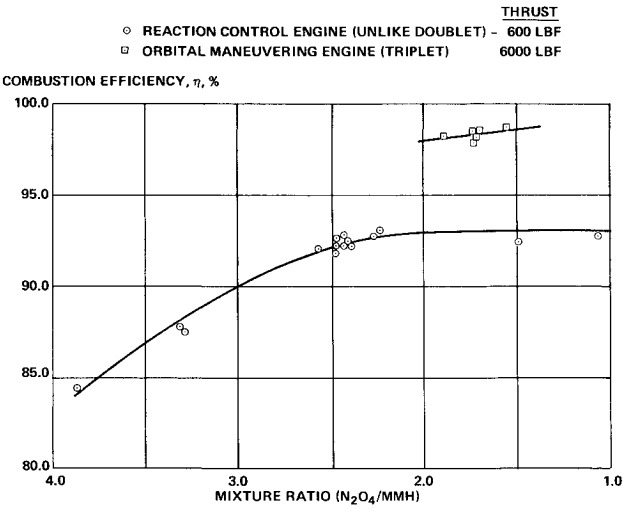

Fig. 8 Liquid/liquid rocket engine performance data.[13,14]

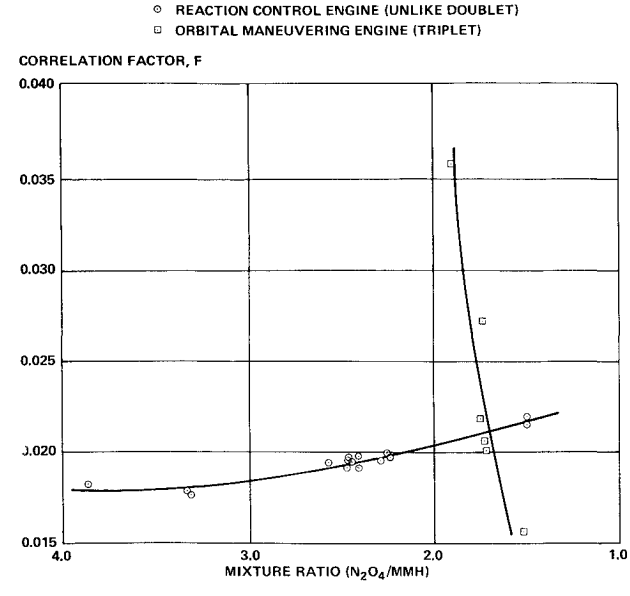

Fig. 9 Correlation of Bell liquid/liquid rocket engines.[13,14]

ply as possible (without *any* modification of the computer program), the calculation of propellant "velocities" were made assuming that the propellants were completely vaporized, so that the injection velocities U_{MMH} and $U_{N_2O_4}$ could be calculated combining the continuity equation and perfect gas law as in Eq. (7), but using appropriate molecular weights. Therefore, the velocities computed in this manner are referred to as "pseudo" velocities, since they are purely fictitious. Of course, the actual injection (liquid) velocities depend on the density of the liquid and not the molecular weight as do gaseous velocities; however, computation of "pseudo" velocities was the simplest way to apply the technique to liquid rocket data. Since the demonstration calculations were made with the same computer program used for the H_2/O_2 propellants, *incorrect* molecular weights (i.e., 2.016 and 32.0) were associated with MMH and N_2O_4 in the η_{pred} computations. Nevertheless, because these computations all were made in a consistent manner, the correlation factor, F, "adjusted" the mixing [Eq. (1)] to the appropriate level for each experimental firing and excellent correlation of all 21 cases was obtained.

The experimental combustion efficiency data, η_{exp}, is plotted in Fig. 8 vs the mixture ratio, N_2O_4/MMH. The mixture ratio was plotted from right to left so that trends would correspond roughly to the previous plots in which V_R was used as the independent variable. Clearly, attempting to correlate such widely scattered data was a considerable challenge for the correlation/prediction technique. Resulting correlations are presented in Fig. 9 in which the correlation factor, F, is also plotted vs the mixture ratio, N_2O_4/MMH. These correlations are unexpectedly good. All the points for each type of injector fall on a smooth continuous curve, suggesting that meaningful predictions may be made for liquid propellants as well as for gaseous propellants. Of course, each of the correlations has very different characteristics because two very different injector configurations are being compared. Naturally, a great deal of additional liquid rocket performance data must be correlated for various types of injectors and ranges of flow conditions before predictions can be made except within the rather limited range of the Bell OME and RCE data.

IV. Conclusions

A simple, semi-empirical performance correlation/prediction technique applicable to gaseous and liquid propellant rocket engines was presented. Correlations were attained by "adjusting" the computation of the gaseous mixing of an unreactive, coaxial jet using a correlation factor, F, which resulted in prediction of the experimental combustion efficiency to within 0.1% for each firing. The nique was successfully applied to Rocketdyne, Aerojet, TRW, and Bell Aerospace gaseous H_2/O_2 rocket engines utilizing coaxial, triplet, trislot, premix, and reverse flow injector elements, and to Bell's 6000 lb-thrust Orbital Maneuvering and 600 lb-thrust Reaction Control engines, which utilize triplet and unlike doublet injector elements, respectively, and liquid monomethylhydrazine and nitric oxide propellants.

The range of conditions over which the gaseous H_2/O_2 rocket engines were correlated is: injector elements 7 to 96, chamber length 3.7 to 8.3 in., L^* 7.7 to 57, chamber pressure 28 to 470 psia, "effective" radius (R_1) 0.026 to 0.71 in., U_{O_2} 90 to 580 fps, U_{H_2} 520 to 3720 ft/sec, V_R 1.04 to 22.6, η_{exp} 81.6 to 99.5%, film-collant level 0 to 30%, O_2/H_2 mass ratio 1.9 to 7,5, F 0.029 to 2.2, and $F \cdot L/R_1$ 3 to 17. For these test data F was essentially independent of η_{exp}, number of injector elements, chamber pressure, and (except in one case) film-coolant level.

Predictions of rocket engine performance using the simple unreactive (cold flow), H_2/O_2 mixing calculation require as input: chamber pressure (P_c), length from injector to nozzle throat (L), number of injector elements (EL), and the mass flow rate, injector area, and total temperature of each propellant. An additional input required is the correlation factor, F, which can be estimated from plots of $F \cdot L/R_1$ vs velocity ratio, V_R, for the injector configuration of interest.

The success achieved with these simple correlations suggests that key rocket design parameters are: 1) injection configuration; 2) chamber length divided by the "effective" radius of the central oxidizer jets, L/R_1; 3) velocity ratio, V_R, (rather than the velocities themselves); and 4) propellant type. The correlation/prediction technique is useful for prediciting optimum operating conditions for a given injector geometry, and for assessing the consistency of test data. Before liquid rocket performance predictions can be made with confidence, additional liquid rocket data, covering a wider range of conditions, must be correlated.

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